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REU PROGRAM AS A STUDENT-CENTERED, PROJECT-BASED ENVIRONMENT FOR TEACHING MATH IN CALIFORNIA STATE UNIVERSITY, CHICO

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Abstract

Starting from 2004, within the last years California State University at Chico (CSUC) hosted the National Science Foundation (NSF) funded Research Experiences for Undergraduates and Teachers (REUT) program in Mathematics. The program targets secondary teachers with a strong interest in mathematics or math education (RET) and students who have completed their junior year with coursework appropriate to the research project (REU). The program is designed to engage participants in research problems with a high potential for publication and to create a research experience that broadens participants' perspective both of mathematics as a discipline and of research (whether mathematical or not) as an exciting exploratory process. During this period the large group of undergraduate students from different US universities and high school math teachers were involved in numerous research projects in pure and applied math, in math education and statistics. Working on the research projects the participants acquire the fundamental mathematical concepts and also studied how to apply modern IT technologies and computational systems for solving various math problems. We provide a research experience that enhances the skills necessary for careers in any of the sciences, while adding both depth and breadth to the participant's mathematical knowledge, skill, and understanding. The depth comes primarily from participation in a research team working on a potentially publishable research problem, while breadth is added by activities that integrate all research teams, by in-service teacher/undergraduate interactions, and by synergistic activities such as weekly research talks. Mathematical activities are supplemented with social activities conducive to creating a productive and comfortable research environment.

Keywords: California State University, Chico, CSUC, Research Experiences for Undergraduates, REU, Research Experiences for Undergraduates and Teachers, REUT, National Science Foundation, NSF.

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ПРОГРАММА REU КАК ОРИЕНТИРОВАННАЯ НА СТУДЕНТОВ ПРОЕКТНАЯ СРЕДА ДЛЯ ПРЕПОДАВАНИЯ МАТЕМАТИКИ В УНИВЕРСИТЕТЕ ШТАТА КАЛИФОРНИЯ В ЧИКО

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Аннотация

В течение последних нескольких лет начиная с 2004 года Университет штата Калифорния в Чико (California State University, Chico — CSUC) становится площадкой для осуществления программы «Исследовательский опыт для студентов и преподавателей» (Research Experiences for Undergraduates and Teachers — REUT) по математике, финансируемой Национальным научным фондом США (the National Science Foundation — NSF). Программа предназначена для учителей средних школ, проявляющих интерес к математике и математическому образованию, и студентов, закончивших первый год обучения с курсовой работой, соответствующей исследовательскому проекту. Благодаря программе REUT ее участники вовлекаются в исследования, результаты которых могут быть в дальнейшем опубликованы в научных изданиях, и получают опыт исследовательской деятельности, который расширяет их представление и о математике как дисциплине, и об исследовании (математическом или ином) как захватывающем процессе. В обозначенный период большая группа студентов бакалавриата из различных университетов США и учителей математики средних школ участвовала в многочисленных исследовательских проектах в области теоретической и прикладной математики, математического образования и статистики. Работая над исследовательскими проектами, участники изучали как фундаментальные математические понятия, так и возможности применения современных информационных технологий и вычислительных систем для решения различных математических задач. Участие в программе дает учителям

и студентам исследовательский опыт, который повышает их компетенции, необходимые для построения карьеры в любой из наук, одновременно добавляя глубину и широту их познаниям в области математики. Глубина исходит прежде всего от участия в исследовательской группе, работающей над проблемой, результаты изучения которой в дальнейшем потенциально могут быть опубликованы, а широта добавляется за счет деятельности, объединяющей все исследовательские группы, за счет взаимодействия преподавателей и студентов без отрыва от их основной деятельности (работы или учебы), а также за счет синергетических мероприятий, таких как еженедельные исследовательские доклады. Математическая деятельность дополняется общественной деятельностью, способствующей созданию продуктивной и комфортной исследовательской среды.

Ключевые слова: Университет штата Калифорния в Чико, программа Research Experiences for Undergraduates, программа REU, программа Research Experiences for Undergraduates and Teachers, программа REUT, Национальный научный фонд США, NSF.

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1. Introduction

The program is designed to engage participants in research problems with a high potential for publication and to create a research experience that broadens participants' perspective both of mathematics as a discipline and of research (whether mathematical or not) as an exciting exploratory process. By doing so, we achieve the following specific objectives:

- encourage undergraduate students, especially those from underrepresented groups, to pursue careers in sciences and engineering, including teaching;
- help to better prepare students to pursue advanced degrees and careers in the sciences;
- provide in-service teachers with a research experience that will foster excitement about mathematics, increase content understanding, and inspire pedagogical innovation in their classrooms;
- promote and enhance mathematical research involving undergraduates at California State University at Chico (CSUC).

Both teachers and undergraduates benefit from working together on a research team. Participants develop connections not only with faculty but also with participants who have very different backgrounds, experiences, and career paths. Students have the opportunity to develop a direct relationship with a math educator. Students also benefit from the experience teachers have in communicating mathematical ideas and may be inspired to consider the possibility of teaching as a career. Secondary teachers benefit from the experience of close contact with outstanding undergraduates and see firsthand what some of their current students will be doing in a few years. Both populations benefit from their complementary mathematical backgrounds which enables them to help one another in their research exploration. A teacher's broader mathematical experience can provide the insight to make a conjecture, while a student's recent exposure to college-level mathematics could provide the specific tools to "make the epsilons and deltas work". CSUC is an ideal location for hosting a combined Research Experiences for Undergraduates and Teachers (REUT). The goals of the REUT mesh closely with CSUC's mission statement, and hence there is strong

institutional support for such a program. In particular, teaching is the institution's primary mission and CSUC embraces a teacher/scholar model in which the main purpose of research is to improve the undergraduate experience. Through the initial grants the participating faculty have gained experience in directing summer research and seen that, not only does this add life to their own research programs, it also benefits our students and department. Inspired by their example, other faculty members are including students in their research projects. Moreover, our REUT plants the seeds for the future by providing our newest faculty members with the chance to work with undergraduates in their first few years at CSUC.

Another guiding mission for CSUC is teacher preparation and support of high school teachers. Many high school mathematics teachers in CSUC's large rural service area already have strong ties to the department and college. The majority of early-career teachers received their credential at CSUC. The university is also a home to the Chico Mathematics Project (CMP), which is charged by the State with providing professional development for in-service mathematics teachers at all grade levels. In addition, CSUC has a Master's program in mathematics education, populated entirely by in-service mathematics teachers who attend the program during three consecutive summers. Several of the fifteen teachers funded through the previous REUT grants have connections with these programs. In fact, three of the fifteen completed Master's theses based on their REUT research and a third is an alumnus of our Master's program. Thus, our REUT fits strongly within the teacher-scholar model and complements and extends the broad range of mathematical activities occurring in Chico during the summer.

2. Nature of Participant Activities

In both academic and industrial positions, scientists and mathematicians are expected to be able to think critically and independently and they are also expected to participate as part of a team. We believe that an REUT experience at CSUC allows participants to hone their own organizational, communication, and critical thinking skills while simultaneously learning how to work as part of a research group. Each summer we organize our REUT participants into research teams,

each of which will include one in-service teacher. Each participant is expected to work both individually and as a contributing member of his or her research team.

Team Activities. We help the participants progress from dependent learners to independent investigators by modeling and explicitly discussing Polya's four stages of problem-solving: understanding the problem, devising a plan of attack, carrying out the plan, and reflecting on the work. Our activities are structured to facilitate the smooth progression from one stage to the next at a pace appropriate to the competence of each participant.

Following that model, in the initial stage, each research team leader offers a mini-course related to the team's mathematical focus area. This allows the team leader to give participants relevant background material, introduce computer software, evaluate the competence of each participant, foster a supportive team environment, and in general ensure the group has the necessary tools to carry out the research project. The team leader concurrently introduces open research problems and students are encouraged to begin their own exploration right away. As a **first step** towards independence, the team selects their research problem. At this point they are given research/expository articles to read in order to better understand their problem. The articles are then discussed in a group setting. These informal discussions help the faculty member ensure that the whole team "understands the problem" and is thus ready for the next stage.

In the **second stage**, the faculty member takes a step back and gradually moves from group director to group member. The team is responsible for "devising a plan of attack", which includes developing research directions and allocating responsibilities. The faculty member contributes to discussions and helps guide the group in fruitful directions without imposing his or her own ideas. He also ensures that each individual is contributing to the development of the team's plan and has a reasonable share of the responsibilities in carrying out the attack. This stage culminates in a presentation of the team's research problem and plan of attack.

In the **third stage**, the faculty member steps back even further. Having helped guide the group in the development of a plan, the faculty member now allows the group to carry out that plan with minimal assistance and acts primarily as an advisor as the team becomes self-sufficient and takes ownership of all aspects of their particular problem. Here, both teachers and undergraduates are confronted with their lack of experience in doing mathematical research. They can help each other to overcome this hurdle and learn to be independent mathematical explorers.

In the **fourth and final stage** the group "reflects" by jointly authoring a written report and preparing a presentation on their research including: a clear statement of their problem, their plan of attack, any obstacles which were encountered, results they obtained, and perhaps directions for further investigation. In this stage the faculty member's role

is primarily to give advice and answer questions that may arise. The reflective stage often continues after the term of the REUT as students work with their team leader to revise a manuscript for publication and prepare to present their research at professional conferences.

Synergistic Activities. Fostering an appropriate research environment is an important part of our program. A successful research team consists of participants who have the following characteristics: they are comfortable expressing their ideas, they aren't afraid of looking foolish they feel they can disagree with other team members, and they know each other. Social interactions are thus an essential component of our REUT program. Space for all participants is reserved at special apartment style dorms on campus to facilitate ongoing unstructured social interaction. Social contacts are further cemented through a weekly series of lectures by prominent invited speakers, each followed by a dinner or reception, and some other social event such as a movie, bowling, or a visit to the Thursday Night Farmer's Market.

We would like our participants to gain not only a deep understanding of a mathematical focus area, but also to develop breadth in their mathematical knowledge and experience. To this end we feel it is important for the three research teams to interact not just socially, but also mathematically. All participants are expected to attend talks given by other participants and the six-week term ends with a full group discussion of the experience.

Within the life-span of the REU program in CSUC we obtained several NSF funded grants. The amount of each grant is around \$300,000 that supports the three years period of the REU activities. Every three years we applied to NSF for funding the extension of our program.

Recently we applied to NSF for funding the next three years term and proposed a number of research projects in different areas of mathematics.

3. Descriptions of proposed research project

1. Number Theory: Euclidean rings of S -integers.

Around 300 B.C., Euclid described an algorithm for computing the greatest common divisor of two integers. The Euclidean algorithm has a myriad of uses, both theoretical and applied. In algebraic number theory, one is interested in other rings of integers which are generalizations of the ordinary integers \mathbb{Z} . One can ask whether these other rings are norm-Euclidean, which roughly means that they possess a Euclidean algorithm [1].

Let $K = \mathbb{Q}(\sqrt{d})$ be a quadratic field. It is a theorem that the ring of integers of K is norm-Euclidean for only finitely many d [2], and those values of d have been completely determined [3, 4]. (Here d is assumed to be square-free, but can be positive or negative.) One expects that inverting a single prime makes it more likely that such a ring will be norm-Euclidean.

It is an open question as to whether inverting a (fixed) finite set of primes leads to infinitely many norm-Euclidean rings of this form. The REUT group would write computer code to gather computational evidence for or against this conjecture. One would like to predict heuristically how often this should occur and ideally prove something about these rings [5]. In a past REUT one team made progress on this problem in the imaginary quadratic case (where $d < 0$) but there are still open questions even in this setting.

There are many related problems one can study; depending upon the interest of the participants these might be of a theoretical or computational nature. One example of a computational problem would be to find examples of rings of S -integers in cubic fields that are norm-Euclidean.

2. Statistics: Integrated system dynamics and agent-based model to address the food, health and environment trilemma.

We will develop a spatial model integrating social science, economic, and technological research to investigate high-level systemic approaches to reducing animal product consumption in order to address the food, environment, and health trilemma. This holistic, systems-based model will assess the impact of different interventions on the total amount of animal products consumed, in order to understand how to best incentivize a plant-forward diet. We will assess the impact of this change on both human health and environmental conditions (e. g., water quality and quantity, air quality, soil quality, and climate). Five types of interventions will be evaluated:

- 1) educational;
- 2) social media and peer influence;
- 3) technological;
- 4) economic policies, particularly in the form of agricultural subsidies and excise taxes;
- 5) regulatory and legal mandates.

“The links between diets, human health, and environmental degradation-known as the diet, health, and environment trilemma-comprise a series of interconnected problems confronting every society globally” [6]. The need to sustainably feed a growing human population is an urgent problem that will only escalate with the effects of climate change. Adopting a plant-forward diet will provide large global environmental benefits while also improving diet related health concerns. A societal shift to a plant-forward diet will reduce diet-related mortality by 17 % and will reduce diet-related environmental impacts by 60 % [6]. Finding solutions that shift diets toward healthier and more sustainable dietary patterns will result in health and environmental benefits on the global scale. Our goal is to establish a shared understanding of food choice dynamics through modeling.

The purpose of the model is to investigate the triggers that move an individual toward a plant-forward diet. We will develop a system dynamics model to evaluate the ability of “top-down” approaches

(government subsidies and policies) to shift individual eating choices. Additionally, we will derive an agent-based model to inform the researchers of the effect of “bottom-up” (educational interventions, social media, and apps) approaches. This integrated hybrid model will allow us to learn what combination of top-down/bottom-up approaches are optimal to influence societal changes in diet choices. The overarching goal of the model is to uncover the social processes involved in shifting to a plant-forward diet and better understand the drivers that cause changes in food choices. The model will also support the identification of intervention strategies and help us understand the social complexity that is involved with food choices. Non-traditional model choices are needed because of the complexity of the food system. Both systems dynamic modeling and agent-based modeling are well-suited to address the complexity of food choice behavior.

Prior to the summer, the current research team will have:

- determined the input and output variables;
- developed the influence diagram;
- developed the flow diagram;
- found the relationships between the variables (mathematical equations, regressions, etc.).

The summer undergraduate research team will analyze and report the model outputs, perform sensitivity analysis, create visualizations, and provide effective non-technical reports.

3. Knot Theory: Knot invariants and traces.

Knot theory is immediately accessible through its physical intuition, yet mysterious and rich with complication. In this project, we will explore the concrete consequences that algebraic structure provides in the study of knots. Many knot invariants, such as the Alexander polynomial, Khovanov homology, and the Temperley-Lieb algebra, arise through the trace and determinant of matrices [7, 8, 9]. Recent advances in algebraic K-theory suggest an overarching framework for such trace-theoretic constructions [10, 11]. By accessing these tools through a simple diagrammatic language, participants will compute knot invariants for a range of examples and examine how the patterns that arise are manifested in the underlying algebra.

The methods involved in this work are visual and syntactic, as is suitable for an undergraduate project. Rather than concentrating on the formalism of the algebra, we will use symbolic methods and computer packages to generate output of the machinery. A primary goal will be to lift known results about classical invariants, such as the symmetry property of the Alexander polynomial and its detection of the perfection of the knot group, to the enriched forms of the invariants arising via algebraic K-theory.

4. Algebra: Special values of Artin-Ihara L -functions.

Special values of L -functions is an active and fertile area of current research in arithmetic geometry. This

field of mathematics can be highly technical though. On the other hand, Stark and Terras [14–16] developed a theory of Artin-Ihara L -functions associated to representations of the group of deck transformations of a Galois cover of multigraphs which is analogous to the usual theory associated to arithmetic objects. It becomes natural to ask if some of the conjectures and theorems about special values of L -functions have analogues for Artin-Ihara L -functions. An analogue of the classical Brumer conjecture, the Stickelberger ideal, and also of a classical theorem of Iwasawa on the l -adic valuation of the class numbers in $\mathbb{Z}l$ -extensions of number fields have been found [13, 17].

The objects involved are much simpler than their counterparts in arithmetic geometry. For instance, Artin-Ihara L -functions are rational functions, and in fact are reciprocal of polynomials. This makes the subject particularly appealing to undergraduate students, since they need less background materials to start doing original research in this area. There are plenty of potential research projects involving Artin-Ihara L -functions. For instance, in the case of function fields over a finite field, the first non-vanishing Taylor coefficient of the zeta functions at $u=1$ is well-understood even when one removes a finite set of places S . It would be interesting to understand a similar phenomenon for multigraphs, since right now, an analogue of S -units is missing. Another interesting question to investigate is to try to introduce ramification. Indeed, Galois covers of multigraphs are unramified everywhere. Such a theory of harmonic morphisms already exists [12], and one wonders if an analogue of the classical Brumer conjecture in this more general setup also holds true. Another possibility of research project would be to extend the results [17], where regular abelian l -covers of bouquets were treated, what if one changes the base graph?

5. Mathematical modeling.

The project contains two parts. The first part is related to mathematical modeling of the subsurface reservoir contamination by the hazardous reactive chemical and/or radioactive materials. In the second part we focus on developing novel models of active bacteria migration in a porous medium.

Part 1. Mathematical modeling of the reactive mass transport in a confined fractured aquifer with porous medium of fractal geometry.

A schematic sketch of the fractured porous aquifer is presented in Figure 1. Cartesian coordinates (x, y) are chosen in such a manner that fluid in the aquifer flows in the x -direction and that the coordinate y is directed upward. It can be readily shown that in the surrounding rocks confining the aquifer from below and above, the gradient of solute concentration in the x -direction is much smaller than that in the direction orthogonal to the aquifer. Since the goal of the present study is to estimate qualitatively the effect of non-Fickian diffusion into the surrounding rocks on the solute transport in the aquifer (rather than to simulate the performance of concrete aquifer within the specific rock formation), it is assumed for simplicity that rocks below and above the aquifer have the same physical properties. The latter assumption makes the process symmetrical with regard to the median line of the aquifer $y = -h$. Therefore, the mass flux equals to 0 at $y = -h$, and the solution of the problem in the sub-domain below this line is identical to the solution in the upper sub-domain. Hence, only the upper half of the domain ($y \geq -h$) can be considered. Let c_2 and c_1 be the concentrations of the solute within the fractured porous aquifer and surrounding porous matrix, respectively. We can assume that porous medium confining the aquifer from the top and the bottom has

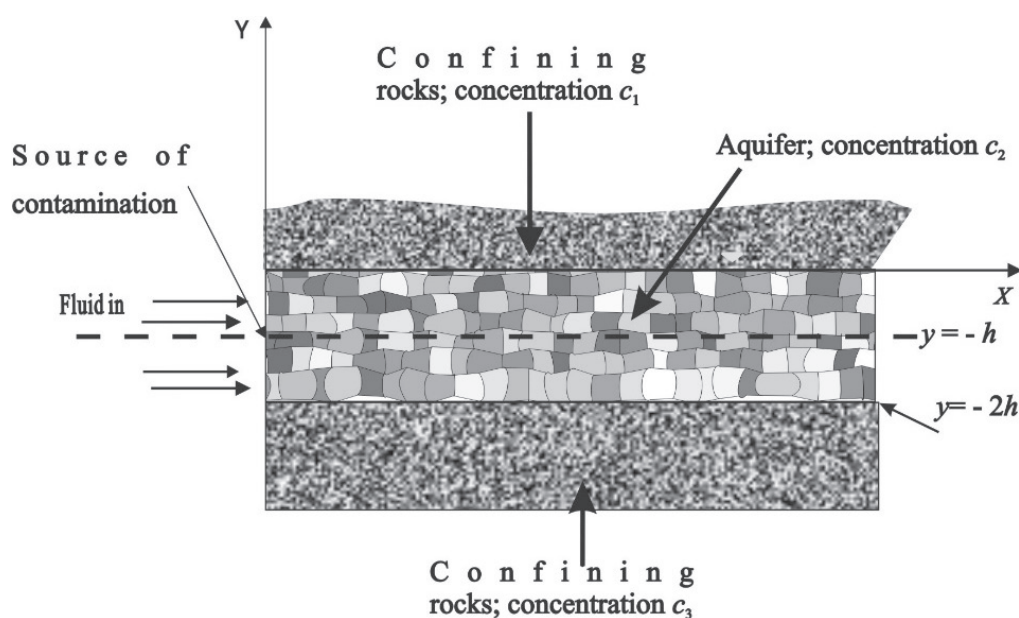


Fig. 1. A schematic sketch of the confined aquifer

a similar fractal structure as the porous blocks within the aquifer.

Below we'll assume (mostly for terminology purposes) that the aquifer is contaminated by the radioactive elements. In the case of any other reactive chemicals, the governing equations are basically the same. Then, due to the above assumptions, accounting for the mass flux continuity condition on the boundaries of the aquifer, and defining the mean concentration of

the solute within this aquifer $c = \frac{1}{h} \int_{-h}^0 c_2 dy$, equations

that model the reactive mass transport in the fracture and porous medium can be presented in the following form:

$$m \frac{\partial c}{\partial \tau} + a \frac{\partial^\mu c}{\partial \tau^\mu} + v \frac{\partial c}{\partial x} = mD \frac{\partial^{\beta+1} c}{\partial x^{\beta+1}} - \frac{1}{h} \frac{\partial s}{\partial \tau} - \lambda c - \lambda \frac{s}{h} + \frac{q}{h}; \quad (1)$$

$$(0 < x < \infty; -h < y < 0; t > 0)$$

$$\frac{\partial^\nu c_1}{\partial \tau^\nu} + \frac{\rho_m}{m^{(1)}} \frac{\partial s_1}{\partial \tau} = D_1 \frac{\partial^{\alpha+1} c_1}{\partial y^{\alpha+1}} - \lambda c_1 - \lambda s_1 \frac{\rho_m}{m^{(1)}}, \quad (2)$$

$$(0 < y < \infty; t > 0)$$

where v is the average velocity of the solution in the aquifer; λ is a radioactive decay constant; τ is time; s is the mass of the solute adsorbed on the walls of the fracture; q is the diffusive mass flux on the aquifer and surrounding rocks interface; D and D_1 are the effective diffusivities in the aquifer and in the surrounding porous medium, respectively, which account for dispersion and molecular diffusion in the fracture and porous medium; s_1 is the mass of the contaminant within the porous matrix, which is adsorbed on the walls of the pores; ρ_m is the density of the rock matrix; m and $m^{(1)}$ are the porosities of the fractured medium and porous medium, respectively; the orders of fractional derivatives ν , α , β and μ are determined by the peculiarities of the distribution of pores and micro-cracks in the blocks and fractal dimension of the medium [18]; the capacity parameter, a , is determined by the properties of the porous medium (such as diffusivities, porosities, fractal dimensions, sizes, etc.) [18]; $\frac{\partial^\alpha c_1}{\partial y^\alpha}$, $\frac{\partial^\beta c}{\partial x^\beta}$, $\frac{\partial^\nu c_1}{\partial \tau^\nu}$, $\frac{\partial^\mu c}{\partial \tau^\mu}$ are the fractional derivatives of the order α , β , ν and μ respectively.

Fractional derivatives can be defined by means of Laplace transformation L . For example spatial derivative $\frac{\partial^\alpha c_1}{\partial y^\alpha}$ can be defined by the equation $L\left[\frac{\partial^\alpha c_1}{\partial y^\alpha}\right] = p^{\alpha-1}(pL[c_1] - c_1(\tau, 0))$, which is equivalent to Caputo definition [18], $\frac{\partial^\alpha c_1}{\partial y^\alpha} = \left[\int_0^y \frac{(y-\xi)^{-\alpha}}{\Gamma(1-\alpha)} \frac{\partial c_1}{\partial \xi} d\xi \right]$.

Inclusion of the fractional derivatives is attributed to the fact that pore geometry and distribution in the rocks is very complex and often resembles the fractal structure. For this type of media the conventional mass

Fickian mass flux can be generalized by introducing the fractional derivative, so that it will take the following

form [18]: $j_1 = -m^{(1)} D_1 \frac{\partial^\alpha c_1}{\partial y^\alpha}$. $j = -mD \frac{\partial^\beta c}{\partial x^\beta}$. Appar-

ently, the mass flux q on the wall of the boundary of the aquifer, $y = 0$, should have the same form, i. e.

$q = -m^{(1)} D_1 \frac{\partial^\alpha c_1}{\partial y^\alpha} \Big|_{y=0}$. The system of equations (1), (2)

will be closed when the relationships between c and s along with c_1 and s_1 are defined. With the good degree of accuracy it can be assumed that $s = K_f c$ and $s_1 = K_m c_1$, where K_f and K_m are given constants.

Various scenarios of the reservoir contamination can be simulated by the proper choice of the boundary and initial conditions. E. g. the following boundary and initial conditions can be imposed for modelling the contamination of the fractured aquifer and surrounding porous rocks in the case when the time dependent source of contamination is located at the inlet of the aquifer

- (a) $\tau = 0, c = c_1 = 0$;
- (b) $x = 0, c = c_0(\tau)$;
- (c) $x \rightarrow \infty, c \rightarrow 0$;
- (d) $y \rightarrow \infty, c_1 \rightarrow 0$;
- (e) $y = 0, c_1 = c$;

where $c_0(\tau)$ is the concentration at the inlet of the aquifer.

Using the technique of integral transforms, solution of the reactive mass transport in the aquifer for various scenarios of contamination will be found analytically. Applying the Mathematica computer algebra to the obtained solution, the process of reservoir contamination will be illustrated graphically.

In this part of the proposed project the following problems will be considered:

- a mathematical model of the reactive contaminant transport in a single fracture (within the fractured aquifer) surrounded by the porous rocks will be derived. Through the use of fractional derivatives, the model will account for contaminant exchange between the fracture and porous rock matrix of fractal geometry and non-local character of chemical reactions within the contaminant;
- a mathematical model of the reactive contaminant transport in a confined, porous, fractured aquifer will be developed and justified. For the case of an arbitrary time-dependent source of contamination located at the inlet of the aquifer, closed-form solutions for radioactive contamination of the aquifer and the confining rock will be obtained and analyzed.

The principal targets of the proposed research are:

- derivation, evaluation and validation of the mathematical model of the contaminant transport in a fractured porous aquifer, which accounts for the fractal geometry of the porous

medium, chemical reactions and/or radioactive contamination;

- numerical and analytical solutions will be obtained and asymptotic analysis of these solutions will be carried out. The adequate values of the model parameters will be proposed and validated by comparison with experimental and field data;
- the effect of chemical reactions (and/or radioactive decay), fractal dimensions of the porous medium, the anomalous character of the diffusion phenomena, and various flow regimes will be analyzed.

Part 2. Mathematical modeling of bacteria migration in a porous medium.

At the Graduate School of Environmental Studies of Tohoku University, Japan, where the PI worked for several years prior to employment with CSUC, the topics related to the application of biotechnology to the environmental problems, such as bioremediation, hydrogen generation by microorganisms, use of carbon dioxide by methane producing bacteria, and removal and decomposition of toxic substances, were of major interest over the last years. During his recent 3-month's visit (summer 2010) to Tohoku University (invited by International 21 Century COE Program in Fluid Dynamics) the PI was involved in discussions regarding the laboratory experiments related to the transport of bacteria in the porous medium, which were carried out at the Laboratory of professor Chihiro Inoue. The key conclusion made within that discussion was that migration of the growing bacteria in the porous media exhibits an anomalous behavior and, therefore, cannot be described by existing traditional models based on advection-diffusion equations coupled with the first type kinetic equation for modeling the absorption-desorption (bacteria attachment-detachment) phenomenon, which are normally utilized for modeling the bacteria transport.

Experimental studies at Tohoku University, Japan were focused on the peculiarities of the bacteria transport by the fluid flow in a porous medium simultaneously with the cell growth (reproduction). Silica sand was packed into the column in which the nutrient solution with suspended cells was injected (Figure 2). The *Lactobacillus casei* (ATCC 15883) was used as a model bacterium. The size of a cell is around 1 μm in diameter and 2 μm in length. Two types of

bacterial cells: growing and resting (due to the lower temperature), were prepared. This allowed testing in different experiments two types of bacteria behavior when the bacteria were active: those rapidly increasing their population, and those not increasing their bacteria population (resting cells). In the laboratory experiment the growth and transport of bacteria was performed in sand with a granule diameter of 600–850 μm and a porosity of 0.4 packed in the tube, whose length $l = 14$ cm and diameter $d = 4.6$ cm. In both cases (growing or resting bacteria) concentration of bacteria in a suspension, at the input c^* was $2.1 \cdot 10^8$ cell/ml and the average flow rate in the porous v medium was 10 cm/hour. The length of the injection period was equal to $t = 0.1$ pore volume ($t = \tau/\tau^*$, $\tau^* = l/v$, l — tube length, v — the suspension mean velocity). Experiments show that the bacteria specific growth rate μ was in the range 0.12–0.16 hr^{-1} . The conventional model for the bacteria transport, based on colloid solution approximation can be presented in the following non-dimensional form

$$R \frac{\partial C}{\partial t} = \frac{1}{Pe} \frac{\partial^2 C}{\partial X^2} - \frac{\partial C}{\partial X} + dC; \quad (4)$$

$$(a) \ t = 0, \ C = 0;$$

$$(b) \ X = 0, \ C = C_0(t); \quad (5)$$

$$(c) \ X \rightarrow \infty, \ C \rightarrow 0,$$

where the following notation is introduced: $C = c/c^*$; $C_0 = c_0(\tau)/c^*$; $Pe = vl/D$; $t = \tau/\tau^*$; $X = x/l$; $D = \alpha_d v$; $\tau^* = l/v$; $d = \mu\tau^*$; c is the concentration of bacteria in the suspension; R is a retardation factor which is bigger than 1 due to the possible sorption of bacteria on the solid matrix; D is a dispersion coefficient which is a linear function of flow velocity v ; μ is the specific growth rate. In equation (4) the first term in the right-hand side accounts for dispersion, the second term — for advection, and the third term is for bacteria growth (reproduction). Equations (5) model the conditions when there aren't any bacteria in the column and describe the concentration variation at the input of the column. Experimental results for the non-active bacteria (bacteria in the rest when $d = 0$) and the results of numerical computations based on the model (4), (5) presented in Figure 3 (a) illustrate that the conventional dispersion-advection model provides the perfect match with the experimental data in this case. However, if bacteria are in an active stage capable for growth and reproduction, the model (4), (5), even though being supplemented with a non-zero term that accounts for the bacteria growth (third term in the right-hand side of (4)), does not provide the adequate description of the bacteria behavior. As it can be seen in Figure 3 (b), for growing bacteria, the experimentally obtained curve of bacteria concentration is shifted to the left from the concentration curve for the resting bacteria. Starting with model (4) and (5), we tried different conventional models that account for trapping and detaching bacteria by trying to calibrate the model by the experimental data by choosing the different values of the controlling

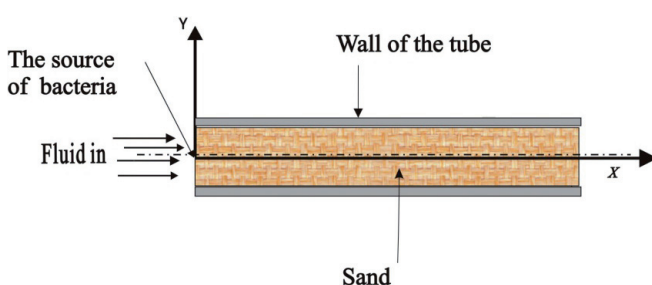


Fig. 2. A schematic of the process (an experimental setup)

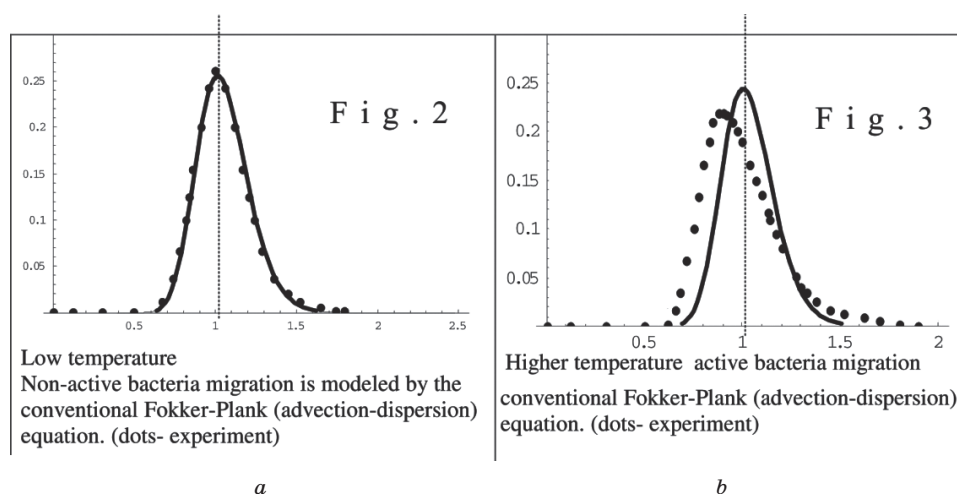


Fig. 3. Experimental results (dots) for passive bacteria (a) and active bacteria (b). Solid line represents the solution of the boundary value problem (4), (5)

parameters (R , Pe , d) for different applications. Our preliminary numerical experiments with the first order kinetic equation of a rate-limited adsorption-desorption process also indicates that this approach does not provide the description of the transport of the growing bacteria matching the experimental data. All these attempts were unsuccessful since the theoretically obtained curve could not be shifted to the left by variation of these parameters. The same attempts were made for the model that in addition to the linear first order non-equilibrium mass transfer equation, which accounts for of the adsorption-desorption, also incorporates variable porosity as a possible result of bio-clogging. It also appeared that this model did not allow moving the theoretically obtained curve for concentration to the left as it was suggested by the experiment. Obviously, the anomalous behavior of the growing bacteria cannot be discussed in terms of classical advection-dispersion models.

One of the modern recently developed mathematical approaches for modeling of various complex chaotic processes (bacteria migration is apparently one of them), is the application of fractional order differential equations. Introduction of the fractional derivatives order is most important for investigation of the reactive processes (growth of bacteria in this case). Our recent advances in application of fractional order differential equations for modeling the anomalous transport of the non-reactive contaminants (see the list of recent publications) allow us to expect that the anomalous transport of growing bacteria can also be well approximated by the models with fractional derivatives. Our preliminary efforts to develop the adequate model based on these modern approaches brought us to the conclusion that the problem of growing bacteria transport should be modeled by accounting for porosity variation due to bio-clogging and for the memory effects in the bacteria transport due to the random character of bacteria trapping and release by the porous matrix. Two phases (mobile and immobile phases) of bacteria existence in the flow should be

considered. Bacteria in a mobile phase have the velocity of the bulk flow whereas bacteria considered in the immobile phase are the bacteria that are trapped by the porous matrix. These bacteria have zero velocity and can clog some of the pores (therefore porosity is not constant). Accounting for the above factors makes the model much more complex. We will denote by c_m and c_i the concentrations of mobile (in the fluid) and immobile (attached to the walls of pores) bacteria, so that the porosity m is a function of the concentration c_i . The conservation equation for c_i can be presented as

$$\frac{\partial c_i m_0}{\partial \tau} = q_{im} + \mu m_0 c_i, \quad (6)$$

where q_{im} is the number of bacteria participating in the exchange between mobile and immobile phases at the unit time, m_0 is the initial porosity, and μ is the effective coefficient of the bacteria population growth. According to Dentz and Berkowitz, 2003, in the sufficiently general case it can be assumed that the correlation between the mobile and immobile phases can be modeled by the following equation:

$$m_0 c_i = \int_0^\tau k(\tau - \xi) m c_m(\xi, x) d\xi, \quad (7)$$

where $k(\tau)$ is the coefficient of bacteria distribution. If $k(\tau)$ is known, then the correlation between the flux of mobile-immobile exchange and the concentration in the suspension, c_m , is defined. If there isn't any bacteria growth $\mu = 0$, but bacteria are alive and can be organized in relatively big families, so that the bio-clogging effect may take place, the equation for the mobile bacteria can be modeled by the equation with the fractional time derivative for complex random processes:

$$\frac{\partial c_m m}{\partial \tau} = K_\alpha \frac{\partial^{1-\alpha}}{\partial \tau^{1-\alpha}} \left[\left(\alpha_d v \frac{\partial^2 c_m}{\partial x^2} - v \frac{\partial c_m}{\partial x} \right) \right], \quad (8)$$

where $\frac{\partial^{1-\alpha} c}{\partial \tau^{1-\alpha}} = \int_0^\tau \frac{(\tau - \xi)^{-(1-\alpha)}}{\Gamma(1 - 1 + \alpha)} \frac{\partial c}{\partial \xi} d\xi$ is the fractional time derivative. It can be readily shown that in this case the

bacteria distribution coefficient will take the following form: $k(\tau) = \frac{A_\beta}{K_\alpha} \frac{\tau^{\beta-\alpha-1}}{\Gamma(\beta-\alpha)} - \delta(\tau)$, where $\delta(\tau)$ is a Dirac delta function. In the case of bacteria growth ($\mu \neq 0$), we have proved that

$$k(\tau) = \frac{A_\beta}{K_\alpha} \frac{\tau^{\beta-\alpha-1} e^{\mu\tau}}{\Gamma(\beta-\alpha)} - \delta(\tau). \quad (9)$$

In this case the equation for the mobile phase concentration should be converted to the following form

$$\frac{\partial c_m m}{\partial \tau} = e^{\mu\tau} \frac{\partial^{1-\alpha}}{\partial \tau^{1-\alpha}} \left[K_\alpha e^{-\mu\tau} \left(\alpha_d v \frac{\partial^2 c_m}{\partial x^2} - v \frac{\partial c_m}{\partial x} \right) \right] + \mu m c_m. \quad (10)$$

After rather tedious evaluations involving the Laplace transform, function q_{im} which defines the number of bacteria transferring from one phase to another can be presented in the following form:

$$q_{im} = \frac{A_\beta}{K_\alpha} e^{\mu\tau} \frac{\partial^{1+\beta-\alpha}}{\partial \tau^{1+\beta-\alpha}} [e^{-\mu\tau} (m c_m)] - \frac{\partial (m c_m) A_\beta}{\partial \tau} + \mu m c_m. \quad (11)$$

The system (9)–(14) completely defines the behavior of active (mobile) and passive (immobile) bacteria in the porous medium. This system should be supplemented by the proper initial and boundary conditions. For example, in the case of modeling of experimental setup illustrated in Figure 1, we can assume that

$$\tau = 0, \quad c_m = c_i = 0, \quad m = m_0; \quad (12)$$

$$x = 0, \quad c_m = c_0(\tau) = \begin{cases} c_0 - \text{const}, & 0 < \tau < \tau_0 \\ 0, & \tau > \tau_0 \end{cases}; \quad (13)$$

$$x \rightarrow \infty, \quad c_m = c_i = 0, \quad m = m_0. \quad (14)$$

The non-linear boundary-value problem (6)–(14) was solved numerically. This solution was calibrated by the data obtained experimentally. Our preliminary results for the one set of available experimental data are presented in Figure 4. As it can be readily seen the solution of the derived model allows us to obtain an exact match of the solution and experimentally obtained data.

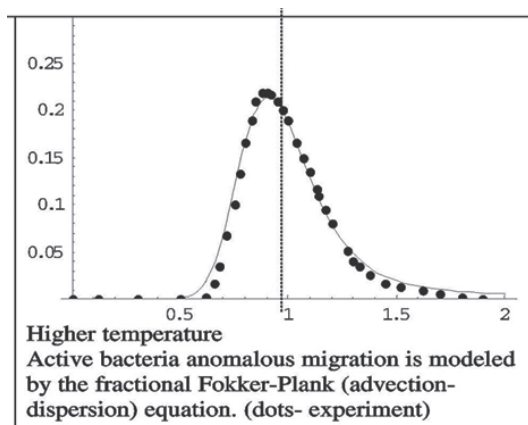


Fig. 4. Experimental results (dots) and numerical solution (solid line) of the problem (6)–(14)

Our preliminary analysis shows that this model can be properly linearized and solved analytically. Using similar approach, which incorporates fractional derivatives, more general models for bacteria migration can be developed. Introducing different forms of the coefficients of bacteria distribution, k , these new mathematical models can simulate the behavior of the various types of bacteria. Several particular cases can be readily specified and solved analytically.

As was already mentioned, increased subsurface pollution, especially due to organic wastes, oil discharges, and leakages from chemical and petroleum plants, is a serious environmental problem for every country. Bioremediation is an effective practical approach based on the application of bacteria to the subsurface systems for eliminating the hazardous waste. Therefore, the proper understanding of the peculiarities of bacteria migration in a porous medium is an important factor for the development and implementation of bioremediation technology. The principal hurdle in empirically developing such understanding is the difficulty and cost of the analysis of bacteria behavior in porous media and water, which severely limits the number of experiments that can typically be implemented. Mathematical modeling of this process can significantly reduce the cost of studies. In this research a quantitative process-based mathematical model capable of describing the fate and transport of growing bacteria in porous media will be developed. Our preliminary studies show that existing mathematical models do not provide an adequate description of the laboratory-observed peculiarities of the process. Mathematical models of the growing bacteria transport based on fractional advection-diffusion equations can be very promising tools for solving the problems of bioremediation of polluted soils. Using this approach, a reliable model of the growing bacteria transport will be developed and validated. This model can be used for predicting the outcomes of the bioremediation technology application to contaminated soils.

4. Intellectual Merit of REUT

In addition to facilitating the solution of research problems in mathematics we provide a model for the integration of research experiences of teachers and undergraduates. The program assessment gauges the success of this model by measuring not only the impact on career decisions of undergraduate participants, but also the major changes in participants' perception of research in general and mathematics in particular. Methods for measuring this impact on participants' mathematical world-view are of interest not only to the NSF, but to science and mathematics educators. As such, this program fits into CSUC's strong tradition of externally funded and innovative programs for teacher preparation and continuing education in mathematics. Moreover, it builds on the department's history of involving undergraduates in research.

5. Conclusion

This project supports the NSF guiding strategy of *advancing discovery and understanding while promoting teaching, training, and learning*. Not only are students involved in research, but also high-school teachers and CSUC faculty are provided with an example of how to integrate research activities into the teaching and learning of math. We provide in-service teachers with a research experience that will foster excitement about mathematics, increase content understanding, and inspire pedagogical innovation in their classrooms. Connections will be built between faculty, students, and teachers by creating new avenues for research activities. Another strategy that is supported through our proposal is to *broaden the participation of under-represented groups*. We encourage our students to pursue advanced degrees in science and engineering. This will help develop a workforce trained to deal with complex problems and increase the number of under-represented individuals in the sciences. We will ensure that at least half of the students supported by renewed REUT funding will be female and at least a quarter from underrepresented minority groups. Additionally, we support the NSF strategy of *enhancing infrastructure for research and education* by promoting faculty research programs at a primarily undergraduate institution. Publications and presentations will be the main method of the *dissemination of our research results*. Our past students have been very active in publishing and presenting REUT research and we expect that tradition to continue. In addition, we encourage our teachers to develop new in-class activities based on their research experience. Our hope is that this will encourage more high school students to pursue careers in STEM disciplines.

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