

FROM THE TEACHING MACHINE MOVEMENT TO INSTRUMENTAL PERSPECTIVE ON TECHNOLOGY-IMMUNE/TECHNOLOGY-ENABLED MATHEMATICS CURRICULUM

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Abstract

The article shares pedagogical ideas about using technology in mathematics teacher education. A point is made that computational power of digital tools may contribute to diminishing cognitive engagement of students in problem solving. In order to avoid such negative impact of technology on one's mathematical development, a new type of mathematics curriculum which integrates the traditional argument and the modern-day computations has been suggested. Examples of technology uses cover all levels of pre-college mathematics and include spreadsheets, Wolfram Alpha, and KidPix.

Keywords: mathematics, problem posing, teacher education, technology, spreadsheets, Wolfram Alpha, KidPix.

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In memory of G. A. Leonov

1. A historical account of using computers in teaching

This article is informed by the author's experience of using computing technology in mathematics education courses for future teachers of North American public schools. The university where the author works is located in the United States about 100 km from Ottawa, and many students enrolled in those courses have been Canadians (residents of Ontario) pursuing their master's degrees in education. The modern focus of mathematics teacher education research and practice, both in the United States and elsewhere, is on the appropriate use of technology (e.g., [1–4]). In the United States, the use of computers in education goes back to the teaching machine movement of the 1920s stemming from the work of Sidney Leavitt Pressey, a professor of psychology at Ohio State University. The major idea behind his work was motivated by the need to support pedagogy of multiple choice questions with “mechanical aids ... [designed to] leave the teacher more free for her most important work, for developing in her pupils fine enthusiasms, clear thinking and high ideas” [5, P. 376]. A Pressey's machine, by providing immediate feedback to a multiple-choice test (mathematics test included), allowed students not only to learn at their own pace but, better still, to become active participants in the learning process [6]. However, only in the second part of the 20th century these pioneering ideas about the power of automated instruction had evolved into the major research and

development agenda, sponsored by the United States Air Force, concerning programmed learning [7].

In mathematical education, the next step in the use of technology was the utilization of the programming languages BASIC and PASCAL, something that enriched mathematics teaching and learning with the flavor of exploration and discovery. A new type of human-computer interaction through writing computer programs emphasized the need for one's grasp of conceptual meaning of the computational procedures involved and their expected outcomes. In turn, this required a combination of conceptual understanding and procedural skills as pillars of mathematical problem solving. Such a shift in mathematics pedagogy towards the unity of the procedural and the conceptual, in addition to teaching mathematics, required, as Demb [8] put it, ‘teaching about’ rather than ‘teaching with’ the computer. Nonetheless, at the end of the 20th century, advances in the development of digital tools reduced the need for computer programming, thus allowing for ‘teaching [mathematics] with’ the computer [9, 10]. Among such advanced computer applications that are relatively user-friendly for mathematics teacher education were various dynamic geometry programs (e.g., *The Geometer's Sketchpad* created by Nicholas Jackiw in the late 1980s), computer algebra systems (e.g., the *Graphing Calculator* (alternatively NuCalc) produced by Pacific Tech [<https://en.wikipedia.org/wiki/NuCalc>]), and electronic spreadsheets (originally developed for uses outside of education). In many cases, a digital tool has become an agent of problem-

solving activities that underlay numeric and/or graphic modeling of mathematical situations under study.

Further evolution of software tools enabled symbolic computations, a traditional part of doing mathematics at different levels of complexity, to be effectively carried out by a computer. For example, in the context of preparing future workforce of the science and technology frontiers, Langtangen and Tveito [11, P. 811–812] noted: “Much of the current focus on algebraically challenging, lengthy, error-prone paper and pencil work can be significantly reduced. The reason for such an evolution is that the computer is simply much better than humans on any theoretically phrased well-defined repetitive operation”. As a result, such technological advances not just opened new research opportunities for proponents of experimental mathematics [12–14], but, quite unexpectedly, created new challenges for mathematics educators. For one, *Wolfram Alpha* (a powerful computational knowledge engine available free on-line at <https://www.wolframalpha.com/>) can answer a question which is linguistically coherent but mathematically ambiguous. Compare (slightly different) wordings of the following two questions: (i) What is *the* second Fibonacci number? (ii) What is second Fibonacci number? The answers to questions (i) and (ii) by *Wolfram Alpha* are, respectively, 2 and 1. That is, the program treats the two wordings differently: the number 2 is the second smallest Fibonacci number and the number 1 is the second term of the sequence 1, 1, 2, 3, 5, Although the wording of question (i) is more linguistically coherent than that of question (ii), in the latter case the program does offer the expected answer. Whereas in the case of Fibonacci numbers of lower ranks the above-mentioned discrepancy has little effect (if any) on problem solving, in the cases of the numbers of higher ranks, this discrepancy may lead to numeric incoherency, especially in the context of problem posing. Moreover, when responding to a question, a computer might offer several options for a student to select an answer. However, one’s ability to navigate through these options often requires knowledge of mathematics beyond the grade level involved. This raises an issue of how curious students can be assisted to use properly what is available when technology affords an easy access to large quantity of information [15]. For example, entering the first five natural numbers into the input box of the On-Line Encyclopedia of Integer Sequences (OEIS®, <https://oeis.org>), a rich source of mathematical knowledge, yields, among other things, its continuation through the numbers 7, 8, 9, 11, 13, ... by referring to sequence A000961 which represents the sequence $2^1, 3^1, 2^2, 5^1, 7^1, 2^3, 3^2, 11^1, 13^1, \dots$ of powers of primes augmented by the number 1.

2. Two types of technology uses in education

Despite all technological innovations available for the teaching of mathematics, the pedagogy of elementary classroom might still be found focusing on drill and practice in arithmetic and basic shape construction in geometry. In the general context of education of the early 1980s, Maddux [16] referred to such perspectives on teaching with computers as Type

I uses of technology and advocated for Type II uses as “new and *better* ways of teaching” (P. 38, italics in the original). Two decades later, a book edited by Maddux and Johnson [17] included diverse classroom experiences specifically reflected on Type II educational uses of technology. In mathematics education, solving multistep problems, using mathematical concepts as tools in computing applications, or exploring curricular topics that otherwise are not grade-appropriate (if not at all unattainable) are the Type II examples.

The notion of the two types of technology application underscores the difference between instructivist and constructivist learning environments that measure learning, respectively, by tests and by one’s ability to conceptualize, brainstorm, conjecture, and discover as a way of grappling with big ideas [18]. Unlike instructivist learning environments that are concerned with a particular way of knowing and knowledge construction, the role of constructivist learning environments is to support multiple ways of knowing in order to help learners develop the so-called epistemic fluency [19] — the ability to comprehend and use different ways of knowing and knowledge construction. Just as the boring and clumsy uses of mathematics could (and should) motivate the development of its more procedurally effective concepts (*e.g.*, repeated addition vs. multiplication), one’s mundane experience with Type I application of technology can motivate the development of creative ideas to support educational applications of Type II.

However, as the sophistication of digital tools progresses, mathematical problem solving with the use of technology, that until recently was considered its Type II educational application, becomes less and less cognitively complex. This reduction in complexity of doing mathematics in the technological paradigm blurs the distinction between the two types. Put another way, a highly sophisticated digital tool enables problem solving to be reduced to a simple push of a button. Furthermore, the type of technology application may depend on what kind of tool is used to support problem solving. Such negative affordance of advanced software products calls for a new mathematics curriculum which allows for both cognitive engagement and technology use as essential elements of mathematical problem solving. That is, the need for cognitively robust problem solving motivates the development of new pedagogical ideas for problem posing.

3. Toward technology-immune/technology-enabled mathematical problems

The growth in symbolic computation capabilities of mathematical modeling software, enabling a student to solve a multistep problem at the push of a button, or, at the very least, making the outcome of problem solving dependent on one’s ability to input all the problem’s data into a computer, something that Guin and Trouche [20] called “an *automatic transport phenomenon*” (P. 205, italics in the original), blurs the dichotomy between the two types of technology uses. In order to continue securing benefits from Maddux’s [16] educational framework, the notion of Type II uses of technology has to be elevated to a higher level where one deals with the so-called technology-

immune/technology-enabled (TITE) problems, the notion originally introduced in [21]. A TITE problem cannot be solved by software at the push of a button, yet technology use in the process of problem solving is critical. Here is an example of a TITE problem geared toward the secondary mathematics education.

Problem 1.

Find a pattern formed by the last digits of the terms of the sequence

$$x_n = \sum_{i=1}^n i^5 \text{ as } n \text{ varies.}$$

A technology-immune part of this problem is to develop (and, as appropriate, prove) the formula $LD(N) = N - 10 \cdot \text{INT}\left(\frac{N}{10}\right)$, where the notations $LD(N)$ and $\text{INT}(N)$ stand, respectively, for the last digit of N and the largest integer smaller than or equal to N . A technology-enabled part of the problem is to use a spreadsheet to model the sequence $LD(x_n)$ for $n = 1, 2, 3, \dots$. Modeling this sequence takes several steps: the first step is to generate (recursively) the sequence of the first 100 (or more) natural numbers, the second step is to generate (recursively) the partial sums of the fifth powers of natural numbers, the third step is to generate the sequence of the last digits of the partial sums by using the formula for $LD(N)$, and the fourth step is to look for a pattern. As a result, one can observe that the sequence of the last digits of the partial sums of the fifth powers of integers forms a 20-cycle $\{1, 3, 6, 0, 5, 1, 8, 6, 5, 5, 6, 8, 1, 5, 0, 6, 3, 1, 0, 0\}$. Furthermore, one can use *Wolfram Alpha* to generate through the formula

`"Table[sum(i^5), i from 1 to n, {n,100}] -
Table[sum((i+20)^5), i from 1 to n, {n,100}]"`

the sequence of numbers each of which has zero as the last digit. One can replace in the above formula the number 20 by a non-multiple of 20, say, by 21, and see the absence of zero as the last digit of any term of the sequence, whatever the value of n . Such technological augmentation of a spreadsheet by *Wolfram Alpha*, a tool capable of symbolic computations, is an example of using an integrated spreadsheet in mathematics education [22].

Remark 1. Whereas finding the last digit of the sum of the first n fifth powers of integers for any given value of n is a task that may be found in an advanced section of a problem-solving book (e.g., [23]), the *appropriate* use of technology not only makes a search for a single digit a more egalitarian task (that is, appealing to more than only students with special interest in mathematics), but it allows for extending the task in order to uncover a 20-cycle formed by the sequence of the last digits. In particular, in the context of a spreadsheet, the word *appropriate* points at an omnipresent overlap of TI and TE parts of the activities. Indeed, even the first step of generating consecutive natural numbers requires conceptual understanding of their recursive development as well as knowledge that a spreadsheet can accommodate a recursive definition through a cell reference technique. Similar TITE problems can be posed for the sums of other powers of integers.

Remark 2. One has to be careful not to use the notation INT in the context of *Wolfram Alpha* because the latter understands the former as integral (antiderivative). So, typing "sum(i^5, i from 1 to 1000)-10*INT(sum(i^5, i from 1 to 1000)/10)" into the input box of *Wolfram Alpha* instructs the program to integrate the sum, yielding something different from its last digit. Instead, one has to use the notation FLOOR . That is, not only difference in traditional (paper-and-pencil) and computer-based techniques can affect the outcome of symbolic computations [24], but mathematical notation may vary from tool to tool. The instrumental perspective on technology-mediated learning (e.g., [25]), a theory developed mostly by French researchers and rooted in Vygotsky's [26] seminal idea of mediation by tools (both material and psychological), distinguishes between an artifact and an instrument in a sense that it is only after one's full understanding of how to use an artifact for a specific task that the artifact becomes an instrument which properly mediates the completion of the task, including mathematical problem solving. In other words, a possible variance in human-computer interaction that depends on denotation calls for one's grasp of conceptual meaning of the procedures involved in computing and their expected outcomes.

4. Looking at the past to develop new teaching ideas in the technological paradigm

In the age of technology, one can use existing mathematical problems as a context for practicing skills in TITE problem posing. Put another way, problem posing can be construed as problem reformulating [27], an activity which is far from being straightforward. A TI part of posing a new problem through reformulating the one already solved consists in the analysis of its solution in order to understand what relationships among the givens make the problem solvable. This kind of analysis, referred to in [28] as turning a doing into an undoing, aims at the development of a conceptual structure common for a family of similar problems. Pedagogically, the need to move from a single problem to a family of like problems is due to the notion (found in the introduction to a book by an Italian educator Maria Montessori) that "often a child left to himself will go back to the same puzzle he solved yesterday, simply for the pleasure of getting it right" [29, P. xxxii]. A TE part of problem reformulation consists in communicating the results of this analysis to a digital tool towards the end of creating a computerized system in which the givens and the answer form the input and the output, respectively. The creation of such a system will be illustrated through the analysis of a historically famous (especially for a Russian reader) problem from the 19th century mathematics curriculum and through the use of the modern-day spreadsheet as a problem-posing tool informed by this analysis.

Education, history, and problem posing can be linked together along the lines of the critical education theory by Paulo Freire, a Brazilian educator. Freire [30] argued that "problem-posing education ... corresponds to the historical nature of humankind ... for whom looking at the past must only be a means of understanding more clearly what and who they are so

cloth. Note that money spent on 138 yards of cloth with the given costs for a yard may vary (assuming only even values) from the smallest sum (one yard of blue cloth, the rest — black), $416 = 3 \cdot 137 + 5 \cdot 1$, to the largest sum (one yard of black cloth, the rest — blue), $688 = 3 \cdot 1 + 5 \cdot 137$.

Also, information presented in Figure 2 can be used to replace in the *Tutor* problem unknowns by knowns and knowns by unknowns. With this in mind, the following problem can be formulated:

Problem 3.

If a merchant spent 500 rubles to buy 80 yards of blue cloth and 50 yards of black cloth, find the price for a yard of each type of cloth.

Noting that $500 = 5 \cdot 80 + 2 \cdot 50$ is the only way to partition 500 into multiples of 80 and 50 brings about the single answer: 5 rubles for a yard of blue cloth and 2 rubles for a yard of black cloth. At the same time, if the above three numbers are replaced by 300 (rubles), 48 (yards), and 6 (yards), the problem would have six integer solutions as

$$\begin{aligned} 300 &= 6 \cdot 48 + 2 \cdot 6 = 5 \cdot 48 + 10 \cdot 6 = 4 \cdot 48 + 18 \cdot 6 = \\ &= 3 \cdot 48 + 26 \cdot 6 = 2 \cdot 48 + 34 \cdot 6 = 1 \cdot 48 + 42 \cdot 6. \end{aligned}$$

6. Formulating TITE problems for young children

As we descend in our examples of TITE problem posing from higher to lower grades, one may ask: How does a TITE problem for a second-grade mathematics classroom look like? What kind of technology can be used by young children in solving a mathematical problem when they are still struggling with addition? Which part of their problem-solving engagement comes first: a TI or TE part? In the author's experience, young children can be observed moving with a relative ease from representing mathematical situation through concrete materials (provided by play-oriented technology) to describing their representations through written symbols. In the context of teaching writing, such transition from visual to symbolic was due to Vygotsky [26] who argued that young children move from using "the first-order symbols ... directly denoting objects or actions ... [to] second-order symbolism, which involves the creation of written signs for the spoken symbols of words ... and develops [by] shifting from drawing of things to drawing of words" (P. 115). Nowadays, kids' drawings as means of problem solving can be enabled by technology. For one, a program *KidPix* (https://en.wikipedia.org/wiki/Kid_Pix) — graphics software used in the schools for various creative activities of young children — offers tools that enable a child in the context of mathematical problem solving to make a shift from using the first-order symbols as forerunners of variables (reflecting quantities associated with the objects of their actions) to the second-order symbolism (reflecting relationships among the quantified objects) as forerunners of algebraic equations. That is, a problem-solving activity of a child begins with a TE part of creating the first-order symbols to act upon, followed by a TI part of developing the second-order symbolism, the appearance

of which is due to cognition with only little residual impact of technology. In turn, problem posing can be activated by teachers' use of the spreadsheet of Figure 2 in order to create didactically coherent mathematical situations to be resolved by young children using trial and error in a play-oriented setting of *KidPix*.

Consider the following problem in which artificial animals called *trimp* and *grimp* are used as forerunners for variables (for more detail, see [32]).

Problem 4.

There are two cages with trimps and grimps in a pet store. In the first cage, among five trimps and two grimps there are 18 legs; in the second cage, among four trimps and three grimps there are 20 legs. How many legs does a trimp have and how many legs does a grimp have?

Figure 3 shows quite an interesting result: a second-grader was capable of implicitly solving a system of two simultaneous equations as resolving a riddle through a physical activity: partition (by trial and error) eighteen legs into five groups of two legs and two groups of four legs, as well as twenty legs into four and three groups of the same cardinality, respectively. Similar to the *Tutor* problem, a partitioning strategy was used as a way of solving a system of two equations in integers. Furthermore, each partition was described by a child through the use of the second-order symbolism in the form of the meaningful to her letters *T* (trimp) and *G* (grimp). Once again, the use of *KidPix* in solving Problem 4 is an example of turning an artifact into an instrument, a process informed by understanding of how the elements of *KidPix* can mediate the transition from the first-order symbols as objects of action to the second-order symbolism as quantitative description of the relationship among the objects.

$$\begin{array}{l} \text{5 T} + \text{2 G} = \text{18 L} \\ \text{4 T} + \text{3 G} = \text{20 L} \end{array}$$

Fig. 3. Solving simultaneous equations in the second grade

Conclusion

The article described the idea of posing and solving technology-immune/technology-enabled problems at three major levels of pre-college mathematics as a way of preparing teachers to teach in the digital era. It was shown how the teaching machine movement, originally aimed at replacing a teacher by a machine in the context of assessment through multiple choice questions, has evolved over the years into a computer-assisted educational system when a seemingly positive affordance of technological sophistication of digital tools created pedagogically challenging conditions allowing for the replacement of a student by a machine. In the case of mathematics, educational problem

solving can become unnecessary computerized thereby leaving a little space for students' concept learning. This motivated the idea of creating new curriculum materials in mathematics to allow for maximizing positive and minimizing negative affordances of the modern-day computers. As a result, the need for a new kind of problems has come to light.

The use of different technological tools in support of technology-enabled parts of the activities was demonstrated in the context of a spreadsheet, *Wolfram Alpha*, and *KidPix*. Depending on grade level, the role of technology as support system for the described activities included computational experiment, problem posing, and play-oriented trial and error problem solving. Theoretically speaking, a TITE problem posing and problem solving can be seen through the lenses of the theory of instrumental genesis when new pedagogical ideas about the appropriation of a digital tool in support of a particular content take shape [20]. In terms of this theory, a technology-immune (TI) part of problem posing and solving may be construed as "instrumentation" — a process through which a subject (*i.e.*, a user of a digital tool) develops intellectually; its technology-enabled (TE) part may be construed as "instrumentalization" — a process through which an artifact (*i.e.*, a digital tool) broadens the realm of utilization due to the user's cognitive growth [33].

In the case of a spreadsheet, the transition from an artifact to an instrument requires basic skills in using cell referencing techniques and spreadsheet formulas. In some cases, the creation of a spreadsheet environment requires rather advanced knowledge of Excel or its appropriate integration with tools of symbolic computation [22]. In the case of *Wolfram Alpha*, the main difficulty in its instrumentation deals with conceptual understanding of the task to be solved by the tool. In the case of *KidPix*, the transition from artifact to instrument requires understanding of how actions on the first-order symbols which comprise the artifact can be interpreted through the second-order symbolism of a specific early algebra task. Moreover, the suggested use of *KidPix* as an artifact, designed originally for kids' entertainment, is a clear case of instrumentation when a non-disciplined creativity of children is advanced to be used within the educational domain of early algebra. Likewise, through learning to replace high school algebra by conceptually-oriented arithmetic teacher candidates grow intellectually and use their instrumentation skills for the creation of a spreadsheet-based (alternatively, artifact-based) computational environment as a self-made instrument which can be used for mathematical problem posing. In that way, the instrumental perspective on TITE mathematics curriculum of teacher education opens a window to new uses of commonly available digital tools to the benefit of diverse learners of pre-college mathematics.

References

1. Conference Board of the Mathematical Sciences. The mathematical education of teachers II. Washington, DC: The Mathematical Association of America, 2012.
2. National curriculum in England: Mathematics programmes of study (updated 16 July 2014). *Department for Education*. Crown copyright, 2014. Available at: <https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study>
3. Standards for preparing teachers of mathematics, 2017. *Association of Mathematics Teacher Educators*. Available at: <https://amte.net/standards>
4. Liu L., Gibson D. (Eds.). Research highlights in technology and teacher education. Waynesville, NC: Association for the Advancement of Computing in Education, 2017.
5. Pressey S. L. A simple apparatus which gives tests and scores — and teaches. *School and Society*, 1926, vol. 23 (586), p. 373–376.
6. Skinner B. F. The technology of teaching. New York: Meredith Corporation, 1968.
7. Lumsdaine A. A. Teaching machines: An introductory overview. In: A. A. Lumsdaine, R. Glaser (Eds.) *Teaching machines and programmed learning: A source book*. Oxford, England: National Education Association, 1960.
8. Demb A. B. Instructional uses of computers in higher education: A survey of higher education in Massachusetts. Paper presented at the EDUCOM Fall Conference (ERIC Document Reproduction Service No. ED 097 028). Toronto, Ontario, 1973.
9. Garofalo J., Drier H., Harper S., Timmerman M. A., Shockey T. Promoting appropriate uses of technology in mathematics teacher preparation. *Contemporary Issues in Technology and Teacher Education* [Online serial], 2000, vol. 1 (1). Available at: <http://www.citejournal.org/vol1/iss1/currentissues/mathematics/article1.htm>
10. Shaw D., et al. Report to the President on the use of technology to strengthen K-12 education in the United States. Washington, DC: President's Committee of Advisors on Science and Technology, Panel on Educational Technology, 1997.
11. Langtangen H. P., Tveito A. How should we prepare the students of science and technology for a life in the computer age? In: B. Engquist, W. Schmid (Eds.) *Mathematics unlimited — 2001 and beyond*. New York: Springer, 2001.
12. Epstein D., Levy S., de la Llave R. About this journal. *Experimental Mathematics*, 1992, vol. 1 (1), p. 1–3.
13. Borwein J., Bailey D. *Mathematics by experiment: Plausible reasoning in the 21st century*. Natick, MA: AK Peters, 2004.
14. Arnold V. I. *Experimental mathematics*. MSRI Mathematical Circles Library, vol. 16. Providence, RI: The American Mathematical Society, 2015.
15. Conole G., Dyke M. What are the affordances of information and communication technologies? *ALT-J: Research in Learning Technology*, 2004, vol. 12 (2), p. 113–124.
16. Maddux C. D. Educational microcomputing: The need for research. *Computers in the Schools*, 1984, vol. 1 (1), p. 35–41.
17. Maddux C. D., Johnson D. L. (Eds.) *Classroom integration of Type II uses of technology in education*. Binghamton, NY: The Haworth Press, 2005.
18. Brooks J. G., Brooks M. G. In search of understanding: The case for constructivist classrooms. Alexandria, VA: Association for Supervision and Curriculum Development, 1999.
19. Morrison D., Collins A. Epistemic fluency and constructivist learning environments. In: B. G. Wilson (Ed.) *Constructivist Learning Environments: Case Studies in Instructional Design*. Englewood Cliffs, NJ: Educational Technology Publications, 1996, p. 107–119.
20. Guin D., Trouche L. The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning*, 1999, vol. 3 (3), p. 195–227.
21. Abramovich S. Revisiting mathematical problem solving and posing in the digital era: toward pedagogically sound uses of modern technology. *International Journal of Mathematical Education in Science and Technology*, 2014, vol. 45 (7), p. 1034–1052.

22. Abramovich S. Exploring mathematics with integrated spreadsheets in teacher education. Singapore: World Scientific, 2016.
23. Linker D., Sultan A. Mathematics problem-solving challenges for secondary school students and beyond. Singapore: World Scientific, 2016.
24. Drijvers P., Trouche L. From artifacts to instruments: A theoretical framework behind the orchestra metaphor. In: M. K. Heid, G. W. Blume (Eds.) Research on technology and the teaching and learning of mathematics, vol. 2 (Cases and perspectives). Charlotte, NC: Information Age Publishing, 2008.
25. Rabardel P., Bourmaud G. From computer to instrument system. *Interacting with Computers*, 2003, vol. 15 (5), p. 665–691.
26. Vygotsky L. S. Mind in society. Cambridge, MA: MIT Press, 1978.
27. Kilpatrick J. Reformulating: approaching mathematical problem solving as inquiry. In: P. Felmer, J. Kilpatrick, E. Pehkonen (Eds.) Posing and solving mathematical problems: Advances and new perspectives. New York: Springer, 2016.
28. Mason J. Asking mathematical questions mathematically. *International Journal of Mathematical Education in Science and Technology*, 2000, vol. 31 (1), p. 97–111.
29. Mayer M. Introduction. In: M. Montessori. The Montessori method. Cambridge, MA: Robert Bentley, 1965.
30. Freire P. Pedagogy of the oppressed (with an introduction by D. Macedo). New York: Continuum, 2003.
31. Tchekoff A. Russian silhouettes (Translated from Russian by M. Fell). Freeport, NY: Books for Libraries Press, 1970.
32. Abramovich S. Early algebra with graphics software as a Type II application of technology. *Computers in the Schools*, 2005, vol. 22 (3/4), p. 21–33.
33. Lonchamp J. An instrumental perspective on CSCL systems. *International Journal of Computer-Supported Collaborative Learning*, 2012, vol. 7 (2), p. 211–237.

О РАЗВИТИИ ВЗГЛЯДОВ НА РОЛЬ ТЕХНОЛОГИЙ В МАТЕМАТИЧЕСКОМ ОБРАЗОВАНИИ: ОТ МАШИНЫ-УЧИТЕЛЯ К ТЕОРИЯМ ИНТЕГРАЦИИ ФОРМАЛЬНЫХ РАССУЖДЕНИЙ И КОМПЬЮТЕРНЫХ ВЫЧИСЛЕНИЙ

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Аннотация

Статья представляет ряд педагогических идей по использованию компьютера в подготовке учителей математики. Высказывается мысль, что вычислительные возможности современных компьютерных технологий могут отрицательно влиять на математическое развитие школьника. В качестве защиты от такого влияния предлагается методика интеграции формальных рассуждений и технологической поддержки для всех уровней школьного образования. Приводятся примеры соответствующего использования спредшитов, Wolfram Alpha и KidPix.

Ключевые слова: математика, постановка задач, подготовка учителей, компьютерные технологии, спредшиты, Wolfram Alpha, KidPix.

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